# EAST AFRICAN FRESHWATER FISHERIES RESEARCH ORGANIZATION 

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## APPENDIX F

## EXAMINATION OF LENGTH FREQUENCY DISTRIBUTIONS TO EVALUATE OVERFISHING

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I originally intended to adapt the single gear approach of BEVERTON and HOLT (1957) to the multiple gear fishery of Lake Victoria. This approach has the limitation of being highly demanding of information, in that it requires growth and natural mortality parameters for each kind of fish. The method of SSENTONGO and LARKIN (1973) for estimating the ratio of the total mortality coefficient $(\mathrm{Z})$ to the van Bertelanffy growth coefficient $(\mathrm{K})$ using length frequency data has been a major step toward using this approach where data are at a minimum, but it is still necessary to get additional information to estimate the absolute levels of the growth coefficient $(\mathrm{K})$ and the natural mortality coefficient (M).

Although the EAFFRO fish tagging programme may eventually provide estimates of $K$ suitable for the Beverton-Holt method, such estimates cannot be expected immediately. I was therefore motivated to find an approach that does not require explicit information on growth or natural mortality. The approach I selected requires only information on average lengths.
it will be assumed that length growth is linear to the maximum length $(\mathrm{L} \infty)$, reached at an age ( t$)$ of unity and that the length-weight relationship is cubic.

$$
\begin{array}{lll}
\mathrm{L}(\mathrm{t}) & =\mathrm{L} \infty \mathrm{t} & \text { when } 0 \leq \mathrm{t} \leq 1 \\
\mathrm{~L}(\mathrm{t}) & =\mathrm{L} \infty & \text { when } \mathrm{t}>1  \tag{1}\\
& & \\
\mathrm{~W}(\mathrm{t}) & =\mathbf{W} \infty \mathrm{t}^{3} & \text { when } 0 \leq \mathrm{t} \leq 1 \\
\mathrm{~W}(\mathrm{t}) & =\mathrm{W} \infty & \text { when } \mathrm{t}>1
\end{array}
$$

This assumption has the advantage of leading to a simple solution, and growth studies by RINNE (1975) have indicated it to hold approximately for Lake Victoria fish. However, the simple growth function is not a necessary feature of my approach, which could be applied to other growth functions, such as the van Bertelanffy equation, requiring however numerical. evaluation of the integrals presented below.

If there is a constant mortality rate $(\mathrm{Z})$, which is the sum of fishing mortality (F) and natural mortality (M), the probability density function of age is

$$
\begin{equation*}
\mathrm{p}(\mathrm{t})=\mathrm{Z}_{\mathrm{e}}^{-\mathrm{Zt}} \tag{2}
\end{equation*}
$$

The weight yield is that fraction of recruitment ( R ) which dies due to fishing multiplied by the average weight $(\overline{\mathrm{W}})$.

$$
\begin{equation*}
\mathrm{Y}=\frac{\mathrm{F}}{\mathrm{Z}} \mathrm{R} \overline{\mathrm{~W}} \tag{3}
\end{equation*}
$$

The average or expected value of weight is

$$
\begin{equation*}
\bar{W}=\int_{o}^{\infty} W(t) p(t) d t \tag{4}
\end{equation*}
$$

Substituting (1) and (2) in (4),

$$
\begin{equation*}
\bar{W}=\int_{o}^{\prime} W \infty t^{3} Z_{e}^{-Z t} d t+\int_{,}^{\infty} W \infty Z_{e}^{-Z t} d t \tag{5}
\end{equation*}
$$

The first term applies to fish which are growing and the second term to fish which are full grown.
Integrating (5),

$$
\begin{equation*}
\bar{W}=W \infty\left[\frac{6}{Z^{3}}-\left(\frac{3}{Z}+\frac{6}{Z^{2}}+\frac{6}{Z^{3}}\right) e^{-z}\right] \tag{6}
\end{equation*}
$$

Substituting (6) in (4)


Note that although $W \infty$ and R must be known to calculate yield they need not be known to identify the fishing intensity at which yield is greatest.

Fig. 1 shows how yield depends upon fishing intensity in equation (7) when natural mortality is unity. The maximum possible yield, which is $12 \%$ of the yield which would be possible if all fish could grow to full size with no mortality before cropping, occurs at a fishing intensity of 0.83 . The yield is severely reduced once fishing intensity is more than three times the optimum.

In order to use equation (7) in practice, it is necessary to estimate $M$ and to translate fishing intensity from practical terms (such as number of boats in the fishery) into F . This can be done from data on average lengths.

The average or expected value of length is

$$
\begin{equation*}
\bar{L}=\int_{0}^{\infty} L(t) p(t) d t \tag{8}
\end{equation*}
$$

Substituting (1) and (2) in (8),

$$
\begin{equation*}
L=\int_{0}^{0} L \infty|+| Z_{e}-Z_{t} d t+\int_{,}^{\infty} L \infty Z_{e}^{-Z t} d t \tag{9}
\end{equation*}
$$

Again, the first term applies to growing fish and the second term to fish already grown.

Integrating (9) and solving for Z ,

$$
\begin{equation*}
\hat{\mathbf{z}}=\frac{\mathbf{L} \infty}{\overline{\mathrm{L}}}\left(1-\mathrm{e}^{-\hat{\mathbf{z}}}\right) \tag{10}
\end{equation*}
$$

Although equation (10) does not have an explicit solution it can be solved easily by interation, using $L \infty / L$ as an initial guess for $Z$.

Because fish of very small sizes may not be properly represented in real samples, it will often be desireable to use fish above only a certain length $\left(L_{o}\right)$ to calculte $L_{0}$. In that case

$$
\begin{equation*}
\mathbf{z}^{\Lambda_{1}}=\frac{\mathrm{L} \propto-\mathbf{L}_{0}\left(1-\mathrm{e}^{-\hat{\mathbf{z}}_{1}}\right)}{\overline{\mathrm{L}}-\mathrm{L}_{0}} \tag{11}
\end{equation*}
$$

$$
\text { and } \hat{z}=\frac{\mathbf{L} \infty_{\mathbf{z}}^{1}}{\mathbf{L} \infty_{0}-L_{o}}
$$

Note that $\hat{Z}$ is not an estimate of mortality per a unit of time such as a year. The time unit is the age at which maximum size is reached, which may not and need not be known.

It is necessary to sample two or more populations experiencing different fishing intensities in order to estimate M. A regression line is fitted with $\mid \hat{Z}_{i}$ as the dependent variable and $f_{i}$, the known fishing intensity in terms of boats or fishing gear, as the independent variable.


$$
\begin{equation*}
\hat{z}_{\mathrm{i}}=\mathrm{M}+\mathrm{b} \mathrm{f}_{\mathrm{i}} \tag{12}
\end{equation*}
$$

The Z-intercept gives an estimate of $M$ (i.e. mortality when fishing intensity is zero) and the slope gives a conversion factor from fishing intensity (f) to F. It is then possible, with M known, to prepare a graph like Fig. 1, position the existing levels of fishing on the graph, and evaluate them with respect to the optimum.

Fish samples taken in Kenya by Wanjala and Marten (presented elsewhere in this report) have demonstrated a striking difference in average length inside and outside of Nyanza Gulf, corresponding to a known difference in fishing densities. During 1975, length frequency information will be collected on all species of fish occuring in bottom trawls. Samples will extend over the entire year to average over seasonal variations in length frequency distributions, and the data on average lengths will be used to calculate Z using equation (11), M using equation (12), and Y using equation (7).

## REFERENCES

BEVERTON, R.J.H. and HOLT, S.J. 1957. On the dynamics of exploited fish populations. HMSO, London. 533p.

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